TURKEY BASTING STACK®

THEORETICAL BACKGROUND

"because I would rather dive head first into my own vomit . . .

Thanksgiving

'91

. . . than eat dry turkey"

KEYWORDS

Thanksgiving \than(k)s-'giv-in\ *n* (1674):

a day appointed for giving thanks for divine goodness: the fourth Thursday in November observed as a legal holiday in the U.S.

turkey \'tər-kē\ *n*:

a large American gallinaceous bird (*Meleagris gallopavo*) that is of wide range in N. America and is domesticated in most parts of the world

cook \'ku̇k\ *vt* :

to prepare food for eating by means of heat

baste \'bāst\ *vt* :

to moisten (as meat) at intervals with a liquid (as melted butter, fat, or pan drippings) esp. during cooking

FUNDAMENTALS

Whenever a temperature gradient exists within a system, or when two systems at different temperatures are brought into contact, energy is transferred.

- THEORY (PRINCIPLES OF THERMODYNAMICS)
  - 1st Law: Energy can neither be created nor destroyed but only changed from one form to another
  - 2nd Law: No process is possible whose sole result is the net transfer of heat from a region of lower temperature to a region of higher temperature
- IMPLICATIONS
  - Turkey basting is accompanied by heat flow resulting from temperature nonequilibrium of the open-oven system
  - Heat transferred during basting MUST be characterized/quantified to permit proper adjustment of turkey cooking time

HEAT TRANSFER

- DEFINITION
  - "The transmission of energy from one region to another as a result of a temperature difference between them"

- MODES OF HEAT TRANSFER
  - Conduction
  - Radiation
  - Convection†

† Strictly speaking, only conduction and radiation should be classified as heat-transfer processes, because only these two mechanisms depend for their operation on the mere existence of a temperature difference. Convection does not strictly comply with the definition of heat transfer because it depends for its operation on mechanical mass transport also. But since we will not be basting in vacuum (this year), and since convection does accomplish transmission of energy from regions of higher temperature to regions of lower temperature, the term "heat transfer by convection" is used without apology.

CONDUCTION

The process by which heat flows from a region of higher temperature (the surface) to a region of lower temperature (the interior) within a medium (the turkey)

- Energy transmitted by direct molecular communication without appreciable displacement of the molecules
- Rate of heat flow by conduction, q_k, in Btu / hr

$$q_k = -k A \frac{dT}{dx}$$

where:

k	=	thermal conductivity of the turkey (Btu / hr ft °F)
A	=	the area of the turkey through which heat flows by conduction, measured perpendicularly to the direction of heat flow (ft^2)
$\frac{dT}{dx}$	=	the temperature gradient (°F / ft)

RADIATION

The process by which heat flows from a high-temperature body (the oven walls) to a body at a lower temperature (the turkey) when the bodies are separated in space -- even when a vacuum exists between them

- Depends upon the *absolute temperatures* and the nature of the body surfaces
- Rate of heat flow by radiation, q_r, in Btu / hr

$$q_r = \sigma A_s \mathcal{F}_{w \to s} (T_w^4 - T_s^4)$$

where:

σ = *Stefan-Boltzman* constant = 0.174×10^{-8} Btu / hr ft^2 °R^4

A_s = the (exposed) surface area of the turkey (ft^2)

$\mathcal{F}_{w-s}$ = a modulus which accounts for the emittances and relative geometries of the the turkey and oven walls

T_w = oven wall temperature (°R)

T_s = surface temperature of the turkey (°R)

CONVECTION

The process of energy transport by the combined action of heat conduction, energy storage, and mixing motion

- Mechanism of energy transfer between a liquid or gas (hot air in the oven) and a solid surface (the turkey)
- Rate of heat flow by convection, q_c, in Btu / hr

$$q_c = \bar{h}_c A_s \Delta T$$

where: h_c = average unit thermal convective conductance (Btu / hr ft^2 °F)

A_s = heat transfer area (ft^2)

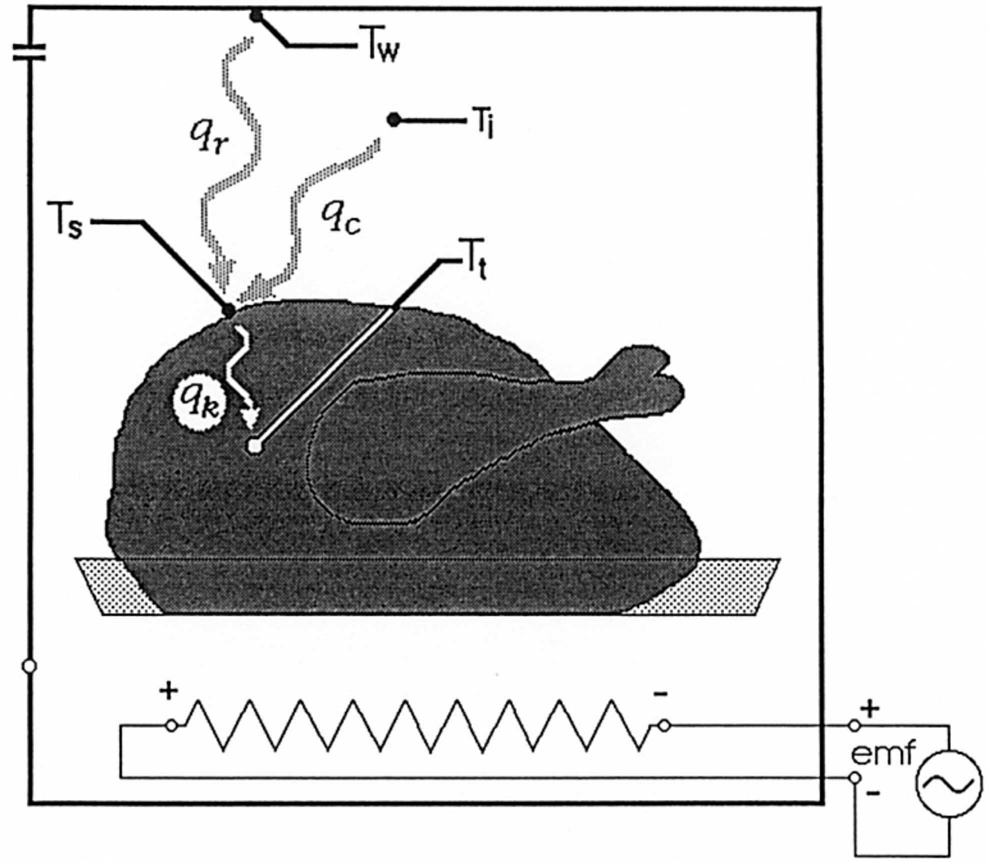
ΔT = difference between the turkey surface temperature T_s and the temperature of the ambient, T_i or T_o (°F)

OVERVIEW

Turkey cooking time, basting time and frequency will be optimized using electrically analogical methods to solve the problems of transient heat flow within the closed- and open-oven-door systems.

Quintessential to these heat transfer analyses are the measurement and acquisition of key system temperatures: turkey meat temperature (T_t), oven wall temperature (T_w), inside and outside oven air temperatures (T_i and T_o, respectively) and room temperature (T_r). Thermal "circuits" are developed and will be used to evaluate the temperature-time history of the turkey surface temperature (T_s). This information will be used in conjunction with measured data to establish the temperature potentials driving the various heat flow processes. An important result from this effort is determination of the heat loss incurred during a baste--as well as an estimate of the additional cooking time required to "recover".

CLOSED-DOOR HEAT TRANSFER



T_t = Turkey Meat Temp

T_s = Turkey Surface Temp

T_w = Oven Wall Temp

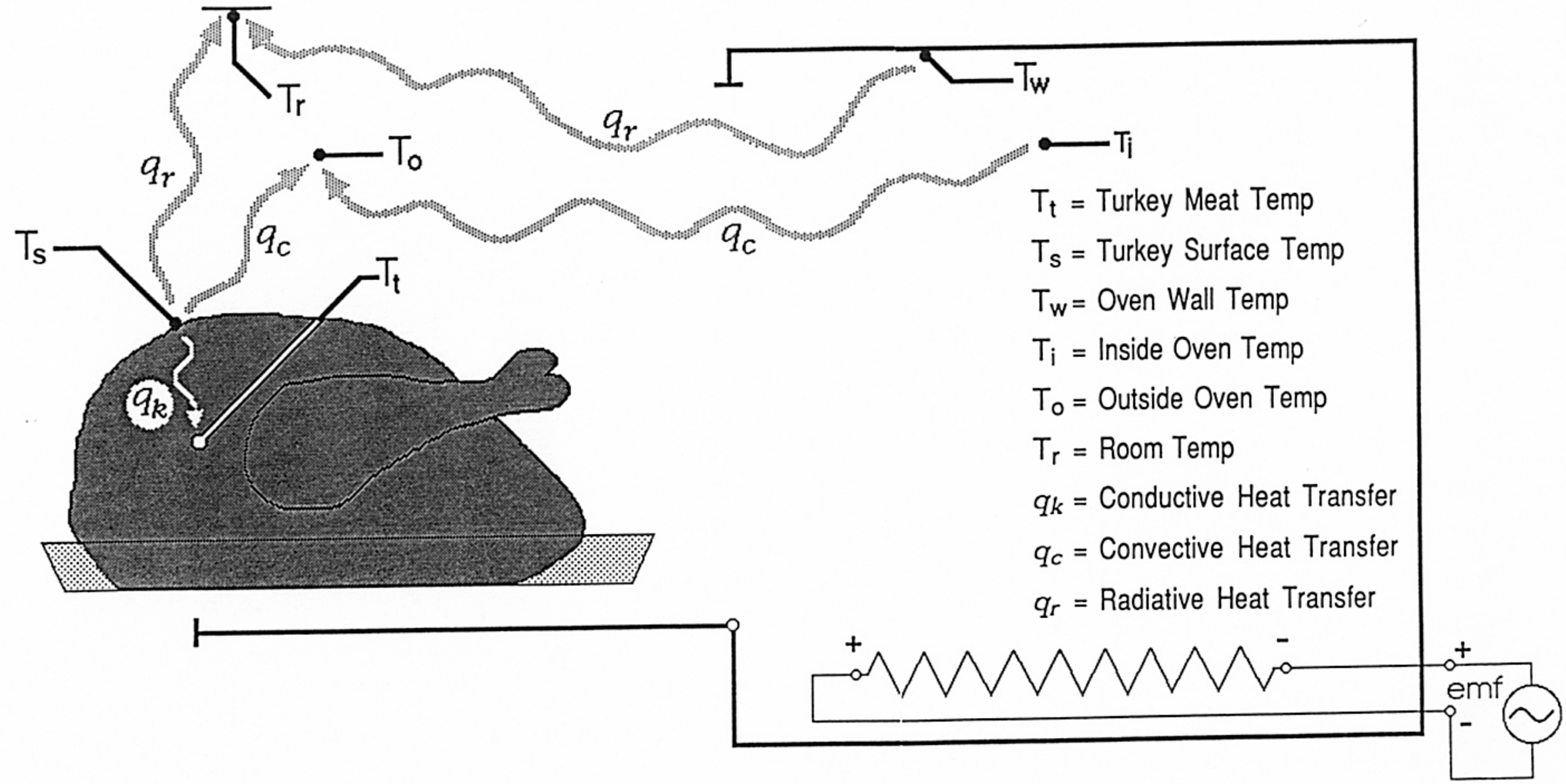
T_i = Inside Oven Temp

q_k = Conductive Heat Transfer

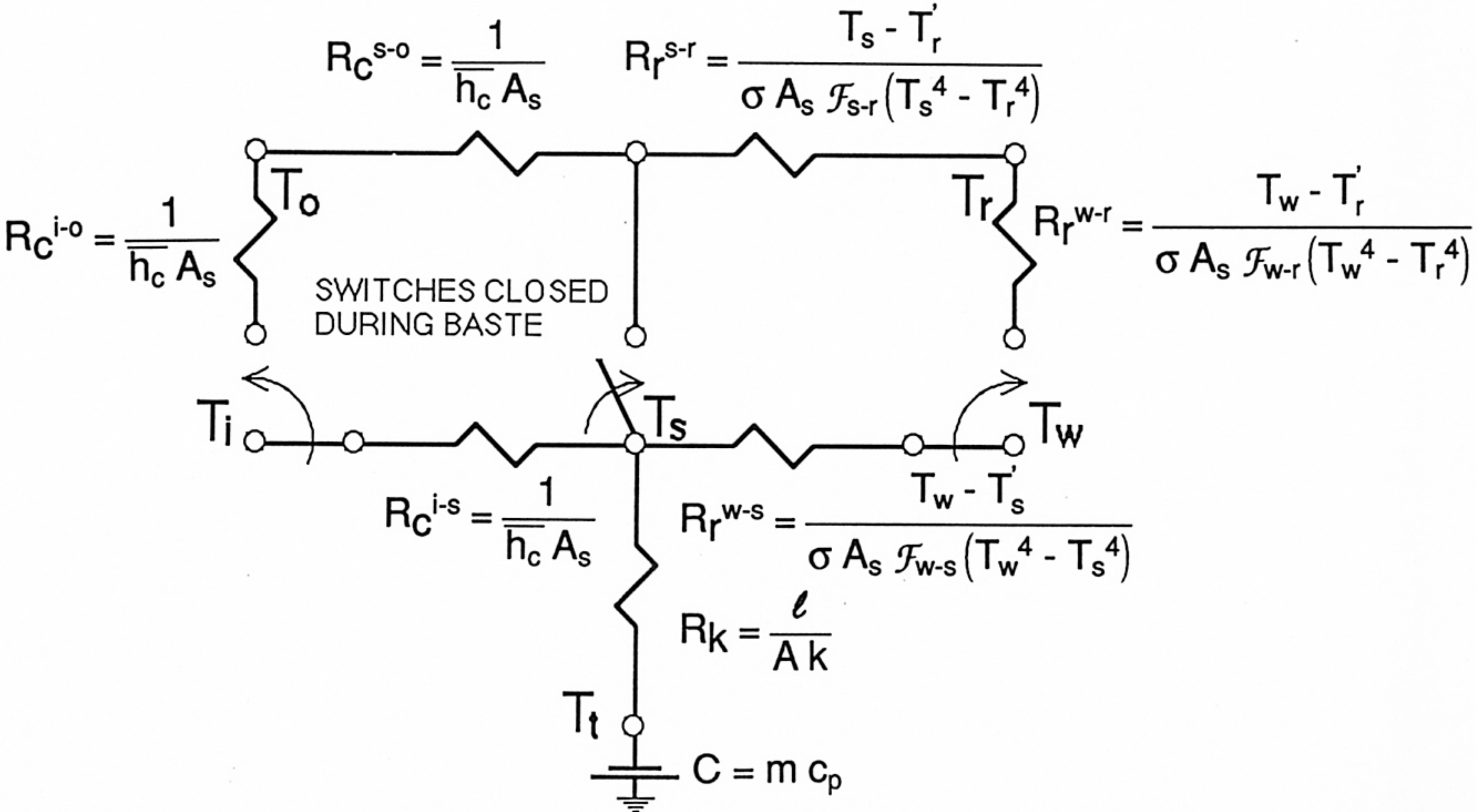
q_c = Convective Heat Transfer

q_r = Radiative Heat Transfer

OPEN-DOOR HEAT TRANSFER



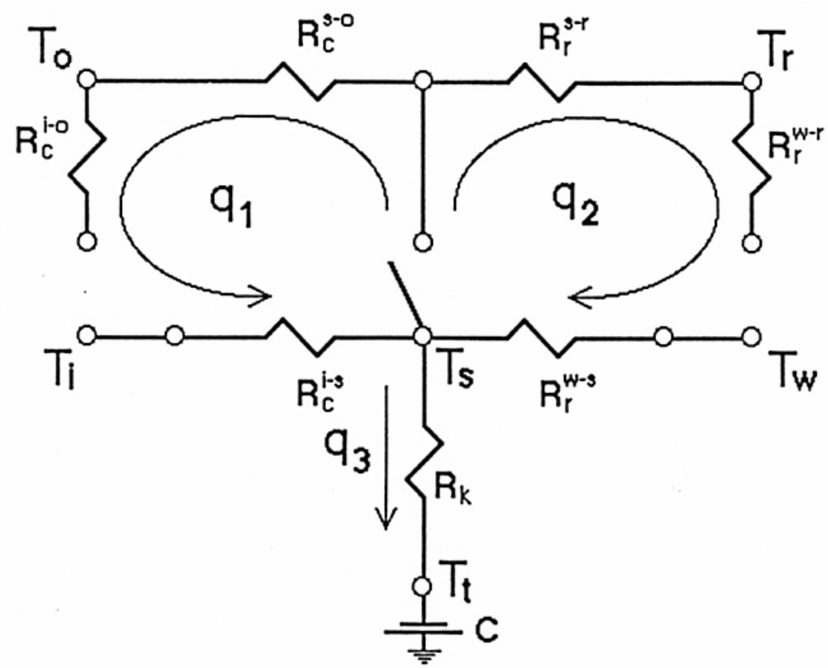
THERMAL CIRCUIT



CLOSED-DOOR CIRCUIT ANALYSIS

ACCORDING TO KIRCHOFF'S FIRST RULE: $q_1 + q_2 = q_3$

i.e.; rate of heat flow from T_i and T_w to T_s = rate of heat flow from T_s to T_t



where:

$$q_1 = \frac{(T_i - T_s)}{R_c^{i-s}}$$

$$q_2 = \frac{(T_w - T_s)}{R_r^{w-s}}$$

$$q_3 = \frac{(T_s - T_t)}{R_k}$$

CLOSED-DOOR CIRCUIT ANALYSIS

$$q1 + q2 = q3 \quad \Rightarrow$$

$$\bar{h}_c A_s \left(T_i - T_s\right) + \sigma A_s \mathcal{F}_{w\text{-}s} \left(T_w^4 - T_s^4\right) = A k \left(T_s - T_t\right) / \ell$$

Solving for T_s:

$$\sigma A_s \mathcal{F}_{w\text{-}s} T_s^4 + \left(\bar{h}_c A_s + A k / \ell\right) T_s$$
$$= \bar{h}_c A_s T_i + A \left(k / \ell\right) T_t + \sigma A_s \mathcal{F}_{w\text{-}s} T_w^4$$

If we define:

$$\alpha = \sigma A_s \mathcal{F}_{w\text{-}s}$$
$$\beta = \left(\bar{h}_c A_s + A k / \ell\right)$$
$$\gamma = \bar{h}_c A_s T_i + A \left(k / \ell\right) T_t + \sigma A_s \mathcal{F}_{w\text{-}s} T_w^4$$

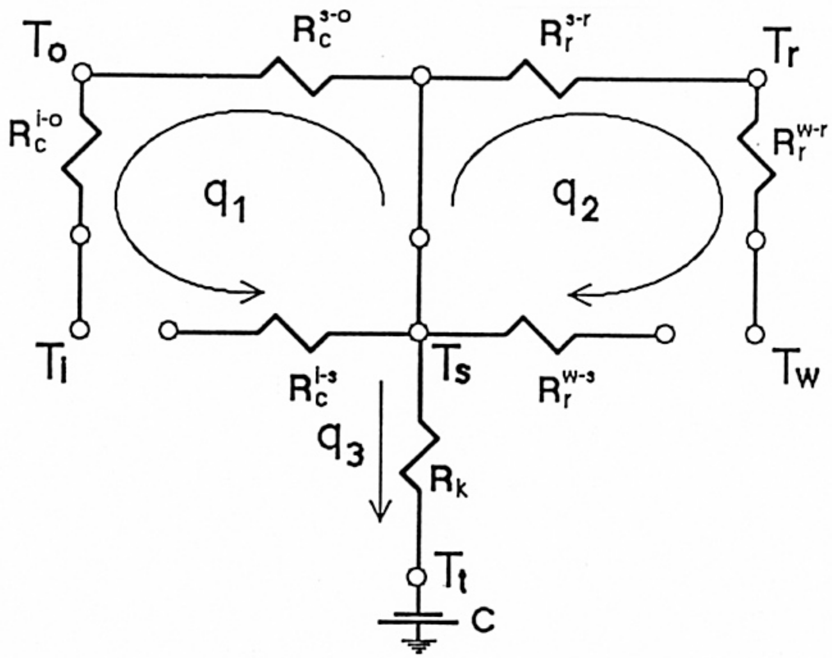
then T_s is obtained as the real, positive root of the fourth order equation:

$$\alpha T_s^4 + \beta T_s - \gamma = 0$$

OPEN-DOOR CIRCUIT ANALYSIS

ACCORDING TO KIRCHOFF'S FIRST RULE: $q_1 + q_2 = q_3$

i.e.; rate of heat flow from T_s to T_o and T_r = rate of heat flow from T_t to T_s



where:

$$q_1 = \frac{(T_s - T_o)}{R_c^{s\text{-}o}}$$

$$q_2 = \frac{(T_s - T_r)}{R_r^{s\text{-}r}}$$

$$q_3 = \frac{(T_t - T_s)}{R_k}$$

OPEN-DOOR CIRCUIT ANALYSIS

$$q1 + q2 = q3 \quad \Rightarrow$$

$$\bar{h}_c A_s (T_s - T_o) + \sigma A_s \mathcal{F}_{s\text{-}r} (T_s^4 - T_r^4) = A\, k (T_t - T_s)/\ell$$

Solving for T_s:

$$\sigma A_s \mathcal{F}_{s\text{-}r} T_s^4 + (\bar{h}_c A_s + A\, k/\ell)\, T_s$$
$$= \bar{h}_c A_s T_i + A\,(k/\ell)\, T_t + \sigma A_s \mathcal{F}_{s\text{-}r} T_r^4$$

If we define:

$$\alpha = \sigma A_s \mathcal{F}_{s\text{-}r}$$
$$\beta = (\bar{h}_c A_s + A\, k/\ell)$$
$$\gamma = \bar{h}_c A_s T_i + A\,(k/\ell)\, T_t + \sigma A_s \mathcal{F}_{s\text{-}r} T_r^4$$

then T_s is obtained as the real, positive root of the fourth order equation:

$$\alpha\, T_s^4 + \beta T_s - \gamma = 0$$

HEAT "LOSS" INCURRED BY BASTING

- Total heat loss penalty, Q_{loss}, equal to the sum of constituent losses, Q_i; *i.e.*,

$$Q_{loss} = \sum_i Q_i$$

- Constituent heat losses computed by integrating--over time--constituent heat flow rates; *i.e.*,

$$q_i = \frac{dQ_i}{dt} \quad \Rightarrow \quad dQ_i = \int dQ_i = \int q_i \, dt$$

CONSTITUENT HEAT LOSS SOURCES

- Convective and radiative heat losses, Q_c and Q_r, respectively--from the turkey to the room--while the oven door is open ($t_{open} \rightarrow t_{close}$); *i.e.*,

$$\left[Q_c + Q_r\right]_{turkey \rightarrow room} = \int_{t_{open}}^{t_{close}} \left[q_c + q_r\right]_{turkey \rightarrow room} dt$$

- Convective and radiative cooking potential loss resulting from transfer of heat from the oven to the room (and resulting in oven temperatures, T_i and T_w, less than 325 °F) ; *i.e.*,

$$\left[Q_c + Q_r\right]_{oven \rightarrow room} = \int_{t_{close}}^{t_{325\,°F}} \left[\left(q_c + q_r\right)_{T_w, T_i = 325\,°F} - \left(q_c + q_r\right)_{actual}\right]_{oven \rightarrow turkey} dt$$

PUTTING IT ALL TOGETHER

$$Q_{loss} = \sum_i \int dQ_i = \sum_i \int q_i \, dt$$

$$\therefore \quad Q_{loss} = \int_{t_{open}}^{t_{close}} \left[\overline{h}_c A_s \left(T_s - T_o\right) + \sigma A_s F_{s \to r} \left(T_s^4 - T_r^4\right)\right] dt$$

$$+ \int_{t_{close}}^{t_{325\,°F}} \left[\overline{h}_c A_s \left\{\left(\left(460+325\right) - T_s\right) - \left(T_w - T_s\right)\right\}\right.$$

$$\left. + \ \sigma A_s F_{w \to s} \left\{\left(\left(460+325\right)^4 - T_s^4\right) - \left(T_w^4 - T_s^4\right)\right\}\right] dt$$

THERMAL CONSTANTS

k = thermal conductivity of the turkey
= 1.2 Btu / hr ft °F

σ = Stefan-Boltzman constant
= 0.174×10^{-8} Btu / hr ft^2 °R^4

$\mathcal{F}_{w\text{-}s}$, $\mathcal{F}_{s\text{-}r}$ = Radiation moduli which account for the emittances and relative geometries between the oven walls→turkey surface and the turkey surface→room
= 0.9, 0.65

$\bar{h}_c$ = average unit thermal convective conductance between the turkey surface and air
= 2.6 Btu / hr ft^2 °F

c_p = specific heat of the turkey
= 0.73 Btu / lb$_m$ °F

m = turkey mass
= turkey weight / (32.2 ft/s^2)

SHAPE APPROXIMATIONS AND FACTORS

- APPROXIMATE THE SHAPE OF THE TURKEY AS THAT OF A *PROLATE SPHEROID*
- CONSIDER AN ELLIPSE WITH SEMI-MAJOR AXIS a, SEMI-MINOR AXIS b, ECCENTRICITY e, CIRCUMFERENCE c, AND PROJECTED AREA p
  - THE FOLLOWING RELATIONS HOLD

$$e = \frac{\sqrt{(a^2 - b^2)}}{a}$$

$$p = \pi a b$$

$$c = 4a\int_0^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2\phi}\, d\phi \approx \pi\left[1.5\,(a+b) - \sqrt{ab}\,\right]$$

  - A *PROLATE SPHEROID* IS FORMED BY ROTATING THE ELLIPSE ABOUT ITS SEMI-MAJOR AXIS

SHAPE APPROXIMATIONS AND FACTORS

- THE SURFACE AREA OF THE *PROLATE SPHEROID* IS

$$S = 2\pi\left(b^2 + a\,b\,\frac{\arcsin e}{e}\right)$$

  - SINCE THE PAN WILL SHIELD A PORTION (SAY 1/3) OF THE TURKEY, WE USE

$$A_S = \frac{2}{3}S = \frac{4}{3}\pi\left(b^2 + a\,b\,\frac{\arcsin e}{e}\right)$$

- THE AREA THROUGH WHICH CONDUCTIVE HEAT IS TRANSFERED GETS SMALLER AS IT APPROACHES THE CENTER OF THE TURKEY, RESULTING IN A NON-LINEAR TEMPERATURE DISTRIBUTION FUNCTION OF THE SEMI-AXES

  - THUS WE DEFINE--AND WILL USE-- A GEOMETRIC MEAN AREA

$$A = \sqrt{A_o A_i}$$, where: A_o, A_i = outside and inside surface areas

$$\ell = r_o - r_i$$ $r_o - r_i$ = radial distance from the outside to the inside (temperature probe)

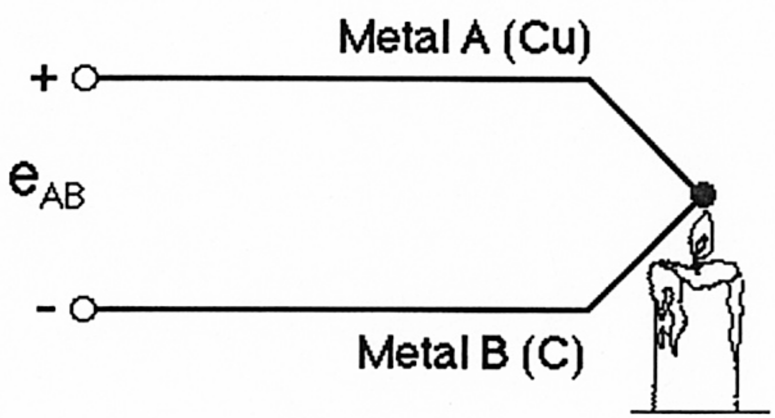
THE THERMOCOUPLE

- WHEN TWO WIRES COMPOSED OF DISSIMILAR METALS ARE JOINED AT BOTH ENDS AND ONE OF THE ENDS IS HEATED, THERE IS A CONTINUOUS CURRENT WHICH FLOWS IN THE *THERMOELECTRIC* CIRCUIT (Thomas Seebeck, 1821)

- IF THIS CIRCUIT IS BROKEN AT THE CENTER, THE NET OPEN CIRCUIT VOLTAGE (the Seebeck voltage) IS A FUNCTION OF THE JUNCTION TEMPERATURE AND THE COMPOSITION OF THE TWO METALS

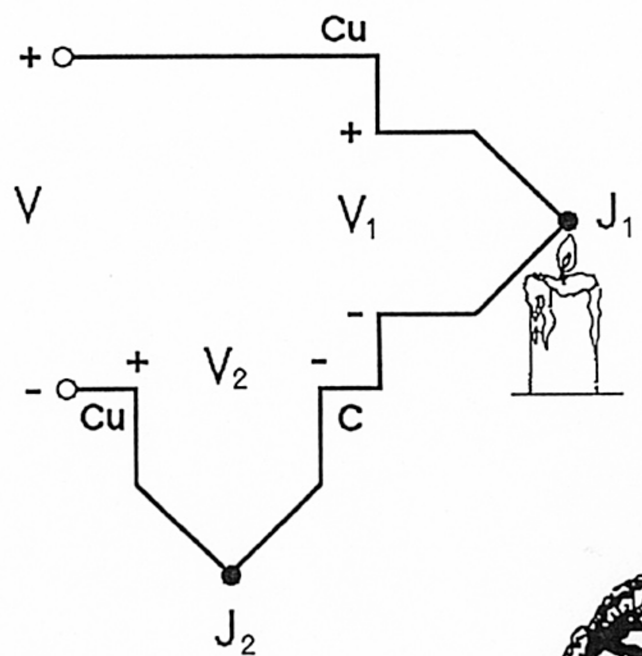
- FOR SMALL CHANGES IN TEMPERATURE THE Seebeck VOLTAGE IS LINEARLY PROPORTIONAL TO TEMPERATURE:

$\Delta e_{AB} = \alpha \, \Delta T$ where: α, the Seebeck coefficient, is the constant of proportionality



MEASURING THERMOCOUPLE VOLTAGE

- WE CAN'T MEASURE THE Seebeck VOLTAGE DIRECTLY BECAUSE WE MUST FIRST CONNECT A VOLTMETER TO THE THERMOCOUPLE, AND THE VOLTMETER LEADS THEMSELVES CREATE A NEW THERMOELECTRIC CIRCUIT
- WE WOULD LIKE THE VOLTMETER TO READ ONLY V_1, BUT BY CONNECTING THE VOLTMETER IN AN ATTEMPT TO MEASURE THE OUTPUT OF JUNCTION J_1, WE HAVE CREATED ANOTHER DISSIMILAR METAL JUNCTION J_2
- THE RESULTANT VOLTMETER READING V WILL BE PROPORTIONAL TO THE TEMPERATURE DIFFERENCE BETWEEN J_1 AND J_2
- THIS SAYS WE CAN'T FIND THE TEMPERATURE AT J_1 UNLESS WE FIRST FIND THE TEMPERATURE OF J_2



MEASURING THERMOCOUPLE VOLTAGE

- ONE WAY TO DETERMINE THE TEMPERATURE OF J_2 IS TO PUT THE JUNCTION INTO AN ICE BATH, FORCING ITS TEMPERATURE TO BE 0 °C AND ESTABLISHING J_2 AS THE *REFERENCE JUNCTION*

- SINCE BOTH VOLTMETER TERMINAL JUNCTIONS ARE NOW COPPER-COPPER, THEY CREATE NO THERMAL EMF AND THE READING V ON THE VOLTMETER IS PROPORTIONAL TO THE TEMPERATURE DIFFERENCE BETWEEN J_1 AND J_2

- NOW THE VOLTMETER READING IS

$$V = (V_1 - V_2) \cong \alpha(t_{J_1} - t_{J_2})$$

IF WE SPECIFY T_{J1} IN °C:

$$T_{J_1}\,(°C) + 273.15 = t_{J_1}$$

$$V = (V_1 - V_2)$$

$$= \alpha[(T_{J_1} + 273) - (T_{J_2} + 273)]$$

$$= \alpha(T_{J_1} - T_{J_2}) = \alpha(T_{J_1} - 0)$$